Multi-Objective Stochastic VRP – Fitness Calculation and Algorithm Converges Using a Generic Genetic Algorithm

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Abstract

Vehicle-routing problems (VRP), which can be considered a generalization of TSP, have been studied in depth. Many variants of the problem exist, most of them trying to find a set of routes with the shortest distance or time possible for a fleet of vehicles. This paper combines two important variants, the stochastic time-dependent VRP and the multi-objective VRP. A genetic algorithm for solving the problem is introduced. A comparison of two fitness functions, with significant difference in computational time, is also presented. Finally, a comparison of solution selection based on TOPSIS method and the two fitness functions is also examined. Results show that a significant decrease in running time, minutes compared to hours, can be achieved, with no impact on the final results of the algorithm.

Keywords: *Multi-Objective, Vehicle Routing, Genetic Algorithms, Fitness, TOPSIS*

I. INTRODUCTION

The Vehicle-Routing Problem (VRP) is a common name for problems involving the construction of a set of routes for a fleet of vehicles. The vehicles start their routes at a depot, call at customers, to whom they deliver goods, and return to the depot. VRP, which can be considered a generalization of the "Traveling-Salesman Problem" [1], is a NP-Hard problem and, therefore, cannot be solved optimally within a reasonable running time. Since the problem was first introduced in 1959, a large number of algorithms for solving it, based on various heuristics and meta-heuristics, as well as extensions to the basic VRP, aiming to produce more realistic models, usually by adding more constraints to the original problem, were introduced. For a discussion about some of the most important algorithms developed so far, and various extensions see [2], [3] and [4].

This paper is based on an on going work, which aims to develop a model and an algorithm for solving the multi-objective real-time vehicle routing problem. In real-time vehicle routing problems, information, such as customers demands, travel time between two points and more, is not known to the algorithm at the beginning, and is reviled as the algorithm progress. If a given solution has to be updated, based on the new information, the changes to the solution are very small (if any) as long as the new information is processed as soon as it has been reviled. Genetic algorithms, a meta-heuristics for solving optimization problems, was chosen as a method for solving the multi-objective real-time vehicle routing problem. In genetic algorithms a set of solution is created in each iteration (the number of iteration is chosen by the user or is defined as a condition), based on the set of solution created in the previous iteration. It is easily possible to update a set of solution based on new information, and continue with this set as the base for the new set of solution. This is the main reason for choosing genetic algorithm. Several factors may affect the quality of a solution obtained from a genetic algorithm. One of them is the quality of accuracy of the fitness functions, on which the algorithm is based when creating the next generation of solutions in a given iteration. In this paper we address the problem of choosing the right fitness function for solving the multi-objective real-time vehicle routing problem, focusing on the accuracy and speed of calculation vs. the quality of the solution and the rate of conversion. To simplify the analysis, instead of using the multi-objective real-time vehicle routing problem, the multi-objective stochastic time-dependent vehicle routing problem, which is a combination of three known extensions, (1) stochastic VRP, (2) time-dependent VRP and (3) multi-objective VRP, was used.

The rest of this paper is as follows. Chapter 2 provides a literature review on Time Dependent VRP, stochastic VRP and multi-objective VRP. Chapter 3 provides a mathematical formulation of the multi-objective stochastic time-dependent vehicle routing problem. A genetic algorithm for solving the problem is described in chapter 3, and a discussion various aspects regarding the algorithm and its performance are presented in chapter 4. The conclusions are presented in chapter 5.

II. LITERATURE REVIEW

A. *Time Dependent VRP*

In the real world, especially in urban areas, the travel time is dependent on both the distance between two customers and the time of day. Ignoring the fact that for some routes the travel time changes throughout the day, may result in solutions that are far from optimal. For that reason, the Time-Dependent VRP (TDVRP) was developed. Whereas most VRP variants look for the shortest paths in terms of length, the TDVRP seeks the shortest paths in terms of travel time.

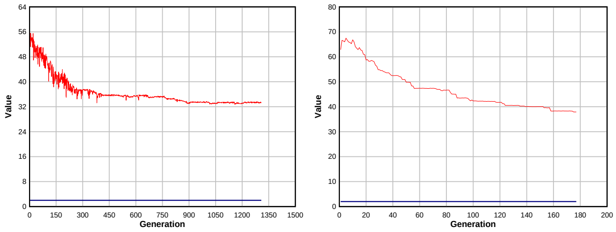

Figure 10. Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem R201 during the first 30 minutes

For problem RC101, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 28.47, objective two equals 9 and objective three equals 0, after 358 generations (see Figure 11). Since the algorithms, when *w*=100, was able to generate 362 generations in 30 minutes, this means that the algorithms best solution was reached after 29 minutes and 40 seconds. For the same problem, RC201, when *w*=1, the algorithm reached the best solution after 798 generations out of 1164 generation that were generated during 30 minutes, meaning, after 20 minutes and 34 seconds. More over, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 25.93, objective two equals 9 and objective three equals 0, after 1099 generations, meaning after 28 minutes and 19 seconds.

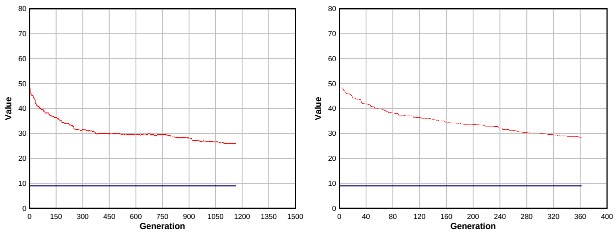

Figure 11. Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem RC101 during the first 30 minutes

For problem RC201, when *w=100* the algorithm, using TOPSIS, reached its best solution in which objective one equals 26.95, objective two equals 2 and objective three equals 0.00002, after 343 generations (see Figure 12). Since the algorithms, when *w*=100, was able to generate 363 generations in 30 minutes, this means that the algorithms best solution was reached after 28 minutes and 20 seconds. For the same problem, RC201, when *w*=1, the algorithm reached the best solution after 314 generations out of 1057 generation that were generated during 30 minutes, meaning, after 8 minutes and 54 seconds. More over, when using *w*=1, the algorithm was able to reach a better solution than the best solution, in which objective one equals 21.69, objective two equals 2 and objective three equals 0, after 930 generations, meaning after 26 minutes and 23 seconds.

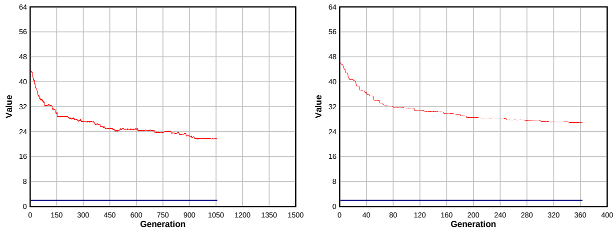

Figure 12. Algorithm convergence when *w*=1 (left) and *w*=100 (right) for problem RC201 during the first 30 minutes

V. CONCLUSIONS

This paper presented the multi-objective stochastic time-dependent vehicle routing problem. An improved version of the VEGA genetic algorithm was also presented. The main problem faced was the fitness functions, which, in order to be accurate, uses simulation, a time consuming operation.

It was shown that it is possible to increase the running time of the algorithm by use an "approximated" fitness function, without influencing the accuracy of the algorithm. A fast algorithm is necessary when coping with real-time problems, which is the final goal.

Two metrics, two set coverage and error ration, were used to compare results obtained from 30 test cases when using an "accurate" fitness function (meaning simulation) and "approximated" fitness function. The results show that there is no difference in the quality of the results obtained using the two methods.

Usually, when solving a multi-objective optimization problem, the result is a set of non-dominated solution, from which, the decision maker has to choose his preferred alternative. Since the final goal is to create an automated algorithm for solving a real-time multi-objective vehicle routing problem, the TOPSIS method, a mechanism for choosing a preferred solution from a set of non-dominated solution has been implemented. It was shown that there is no difference in the quality of the results obtained using the "approximated" or "accurate" methods, however, this does not mean that the same results exist in both sets, and therefore it is not guarantied that the TOPSIS method selects similar results from both sets. It was shown, by means of correlation testing and paired-samples t-tests, that the solutions selected by the TOPSIS methods are similar regardless of the method used for calculating the fitness functions.

Since travel time is more likely to be lognormally distributed a second set of tests was done, using Solomon's instances. Using 500 generation and a population of 200 chromosomes, the result of the IVEGA algorithm showed that for problems with large number of chromosomes (50 and 100 customers) using *w*=100 results with a better solution the when using *w*=1, while for problems with small number of customers (25 and 50) no significant difference was found. Since it is known that the number of generations used by a genetic algorithm may affect its results, and since in real-time applications, the number of generations is bounded by the time given to the algorithm to come with a solution. The algorithm was tested again, this time the stopping condition was 30 minutes of running time, instead of the 500 generations. This time the result showed that in all cases, the result obtained by the algorithm when *w*=1 are better then the results obtained when *w*=100. Moreover, when *w*=1, the algorithm converges to the best solution much faster than when *w*=100.

A future research should test more cases, in order to broaden the analysis. More metrics can be used for the comparison of the results obtained by using the different methods for calculating the fitness functions.

Furthermore, it is possible to add more objectives to the problem, and check whether the results remain the same, or whether the results shown in the paper are specific to the selected objectives.

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